4. V. A. Borodulya, Yu. A. Buevich, and V. V. Zav'yalov, "Stability of operation of units with a granular bed fluidized by a gas flow," Inzh.-Fiz. Zh., 31, No. 3, 410-417 (1976).
5. Yu. A. Buevich, A. F. Ryzhkov, and N. M. Kharisova, "Instability of a vibrofluidized bed," Inzh.-Fiz. Zh., 37, No. 4, 626-634 (1979).
6. A. K. Mikhailov and V. V. Malyushenko, Rotodynamic Pumps (Theory, Design, Construction) [in Russian], Mashinostroenie, Moscow (1977).
7. S. L. Sau, Hydrodynamics of Multiphase Systems [Russian translation], Mir, Moscow (1971).
8. A. A. Andronov, A. A. Vitt, and S. E. Khaikin, Theory of Vibrations [in Russian], Nauka, Moscow (1981).
9. V. A. Glinskii, I. O. Protod'yakonov, and Yu. G. Chesnokov, "Experimental study of the rate of rise and growth of bubbles in a fluidized bed by the method of x-ray photography," Zh. Prikl. Khim., 54, No. 1, 183-184 (1981).
10. J. S. Bendat and A. G. Piersol, Random Data: Analysis and Measurement Procedures, Wiley (1971).

CALCULATION OF MASS TRANSPORT IN NONISOTHERMAL EVAPORATION OF LIQUIDS FROM CAPILLARIES WITH CONSIDERATION OF VARIABLE VISCOSITY OF THE VAPOR–GAS MIXTURE

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The principles of vapor transport in a cylindrical capillary with temperature gradient are studied. Expressions are found for vapor flux and pressure of the mixture above the liquid meniscus in various evaporation regimes.

We will consider a cylindrical capillary of radius r, filled by a liquid, from the open surface of which evaporation occurs. We direct a coordinate axis from the mouth of the channel ($x = 0$) toward the liquid meniscus ($x = l$). We assume that the partial vapor pressure at the channel mouth P_{01} is constant and always less than the saturated vapor pressure at the liquid meniscus temperature $P_S[T(\ell)]$. The temperature varies along the capillary axis linearly, $T(x) = T_0 + \nabla Tx$. The binary gas mixture into which the liquid evaporates consists of molecules of vapor (first component) and gas (second component). We will perform the analysis with the assumption that the medium is continuous ($Kn \ll 1$) and that the flow of the vapor–gas mixture within the capillary is steady-state and one-dimensional. Thermodiffusion and barodiffusion components of the flow will not be considered because of their smallness.

In the general case in which no limitations are imposed on the vapor transport regime and it is necessary to consider both the hydrodynamic flow of the vapor–gas mixture and interdiffusion of the components, the densities of the steady-state vapor and gas flows in a coordinate system fixed to the capillary are described by the following equations:

$$j_1 = -D(x)\frac{d\rho_1(x)}{dx} - \frac{r^2\rho_1(x)}{8\eta(x)}\frac{dP(x)}{dx} = \text{const}, \tag{1}$$

$$j_2 = -D(x)\frac{d\rho_2(x)}{dx} - \frac{r\rho_2(x)}{8\eta(x)}\frac{dP(x)}{dx} = 0. \tag{2}$$

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The vapor flow density through any section of the cylindrical capillary is constant. This follows from the continuity equation when the transport process is considered in the steady-state approximation. Equality to zero of the gas flow density is a consequence of the condition of its nonpenetration through the boundary separating the phases.

The diffusion coefficients for vapor molecules into the gas and gas into the vapor are assumed equal and their dependence on total mixture pressure and temperature have the form (1)

$$D_{12}(x)=D_{21}(x)=D(x)=\frac{D_0P_0}{P(x)}\left[\frac{T(x)}{T_0}\right]^m, \tag{3}$$

where $P_0 = P(x = 0)$; $D_0 = D(x = 0)$; m is a constant number.

Each of the mixture components obeys the ideal gas law, so that

$$\rho_1(x)=\frac{\mu_1P_1(x)}{RT(x)},\quad \rho_2(x)=\frac{\mu_2P_2(x)}{RT(x)}. \tag{4}$$

In the majority of cases the dynamic viscosity coefficient of the mixture can be determined accurately by the following empirical expression (2):

$$\eta(x)=\left[\frac{P_1(x)}{P(x)}\eta_1+\frac{P_2(x)}{P(x)}\eta_2\right]\left\{1+\frac{1}{a}\left[\frac{P_1(x)}{P(x)}-\frac{P_1^2(x)}{P^2(x)}\right]\right\}. \tag{5}$$

Considering the relationship $P_1(x) + P_2(x) = P(x)$, we obtain from Eq. (5)

$$\eta(x)=\eta_1\left[1+\left(\frac{\eta_2-\eta_1}{\eta_1}+\frac{1}{a}\right)\frac{P_2(x)}{P(x)}+\frac{(\eta_2-2\eta_1)P_2^2(x)}{a\eta_1P^2(x)}-\frac{(\eta_2-\eta_1)P_2^3(x)}{a\eta_1P^3(x)}\right], \tag{6}$$

where η_1 and η_2 are the viscosities of the components at the mean temperature of the vapor–gas mixture $T(\ell/2) = T_0 + \nabla T\ell/2$; a is some constant coefficient (for a mixture of water vapor with air $a = 2.75$).

With consideration of Eq. (4), Eqs. (1) and (2) reduce to the form

$$j_1=-\frac{\mu_1D(x)}{RT(x)}\left\{\frac{dP_1(x)}{dx}-\frac{\nabla TP_1(x)}{T(x)}+\frac{r^2P_1(x)\,dP(x)}{8\eta(x)D(x)\,dx}\right\}, \tag{7}$$

$$j_2=-\frac{\mu_2D(x)}{RT(x)}\left\{\frac{dP_2(x)}{dx}-\frac{\nabla TP_2(x)}{T(x)}+\frac{r^2P_2(x)\,dP(x)}{8\eta(x)D(x)\,dx}\right\}=0. \tag{8}$$

With the aid of Eqs. (7), (8) we determine the vapor flux density

$$j_1=-\frac{\mu_1D(x)}{RT(x)}\left\{\frac{dP(x)}{dx}-\frac{\nabla TP(x)}{T(x)}+\frac{r^2P(x)\,dP(x)}{8\eta(x)D(x)\,dx}\right\}, \tag{9}$$

or, according to Eq. (8),

$$j_1=-\frac{\mu_1D_0P_0}{RT_0\left(1+\dfrac{\nabla Tx}{T_0}\right)^{1-m}}\,\frac{d\left[\ln\dfrac{P(x)}{P_2(x)}\right]}{dx}. \tag{10}$$

We obtain an expression for the vapor flux density by integrating this last equation with the known boundary conditions $x = 0$, $P(x) = P_0$, $P_2(x) = P_0 - P_{01}$; $x = \ell$, $P(x) = P_\ell$, $P_2(x) = P_\ell - P_s(\ell)$:

$$j_1=-\frac{\nabla T(2-m)D_0\mu_1P_0}{\left[\left(1+\dfrac{\nabla Tl}{T_0}\right)^{2-m}-1\right]RT_0^2}\ln\frac{P_l(P_0-P_{01})}{P_0[P_l-P_s(l)]}. \tag{11}$$

The coefficient $\nabla T\ell(2 - m)/T_0[1 + \Delta T\ell/T_0)^{2-m} - 1]$ in this equation considers the effect on the intensity of vapor transport of change in temperature and the diffusion coefficient over channel length, beginning from the mouth. If the condition

$$\nabla Tl<\frac{T_0}{2} \tag{12}$$

is satisfied, this coefficient can be calculated with an uncertainty of no more than 0.5% while retaining only the first three terms of the expansion of the function $(1 + \Delta T\ell/T_0)^{2-m}$ in a power series. In this case Eq. (11) simplifies:

$$j_1 = -\frac{D_0\mu_1 P_0\left(1+\dfrac{\nabla Tl}{2T_0}\right)^{m-1}}{RT_0 l}\ln\frac{P_l(P_0-P_{01})}{P_0[P_l-P_s(l)]}. \tag{13}$$

With increase in ℓ the saturated vapor pressure above the liquid meniscus changes, as follows from the Clapeyron–Clausius equation, by the law [3]:

$$P_s(l) = P_s(0)\exp\left\{\frac{L_0\nabla Tl}{RT_0T(l)}\right\}. \tag{14}$$

In Eq. (13) the value of the vapor–gas mixture pressure above the liquid meniscus P_ℓ remains unknown. From Eq. (8) with consideration of Eqs. (5), (3) we have

$$\frac{dP_2(x)}{P_2(x)} + \left(\frac{\eta_2-\eta_1}{\eta_1}+\frac{1}{a}\right)\frac{dP_2(x)}{P(x)} + \frac{(\eta_2-2\eta_1)P_2(x)\,dP_2(x)}{a\eta_1P^2(x)} - $$
$$-\frac{(\eta_2-\eta_1)P_2^2(x)\,dP_2(x)}{a\eta_1P^3(x)} - \frac{\nabla T dx}{T(x)}\left\{1+\left(\frac{\eta_2-\eta_1}{\eta_1}+\frac{1}{a}\right)\frac{P_2(x)}{P(x)} + \right.$$
$$\left. + \frac{(\eta_2-2\eta_1)P_2^2(x)}{a\eta_1P^2(x)} - \frac{(\eta_2-\eta_1)P_2^3(x)}{a\eta_1P^3(x)}\right\} = -\frac{r^2P(x)\,dP(x)}{8\eta_1D_0P_0\left(1+\dfrac{\nabla Tx}{T_0}\right)^m}. \tag{15}$$

To find the dependence of the molar fraction of gas in the mixture on coordinate we integrate Eq. (10) with boundary conditions $x = 0$, $P_2 = P_0 - P_{01}$, $P = P_0$; $x = x$, $P_2 = P_2(x)$, $P = P(x)$. As a result, we obtain an expression for the vapor flux density:

$$j_1 = -\frac{\nabla T(2-m)D_0\mu_1P_0}{\left[\left(1+\dfrac{\nabla Tx}{T_0}\right)^{2-m}-1\right]RT_0^2}\ln\frac{P(x)(P_0-P_{01})}{P_0P_2(x)}. \tag{16}$$

We will equate the right sides of Eqs. (11) and (16), with consideration of condition (12):

$$\frac{P_2(x)}{P(x)} = \frac{(P_0-P_{01})}{P_0}\exp\left\{-\frac{x}{l}\ln\frac{P_l(P_0-P_{01})}{P_0[P_l-P_s(l)]}\right\}. \tag{17}$$

If the meniscus temperature is less than the liquid boiling point, then $P_s(\ell) < P_0$ and for practically any capillary radius P_ℓ differs only insignificantly from P_0, so that in Eq. (15) we may consider $P(x)$ constant and equal to a mean value $P(x) = (P_0 + P_l)/2$. If the meniscus temperature is above the boiling point, then $P_s(\ell) > P_0$ and P_ℓ is insignificantly greater than $P_s(\ell)$ and with no significant reduction in accuracy we may take $P(x) = [P_\ell + P_s(\ell)]/2$ in Eq. (15). Integrating Eq. (15) from the channel mouth to the meniscus with the boundary conditions indicated above and considering Eq. (17), we obtain a transcendental equation for calculation of the mixture pressure above the liquid meniscus:

$$\ln\frac{P_l-P_s(l)}{P_0-P_{01}} + \left(\frac{\eta_2-\eta_1}{\eta_1}+\frac{1}{a}\right)\frac{2(P_l-P_s-P_0+P_{01})}{(P_l+P)} + \frac{2(\eta_2-2\eta_1)[(P_l-P_s)^2-(P_0-P_{01})^2]}{a\eta_1(P_l+P)^2} - $$
$$-\frac{8(\eta_2-\eta_1)[(P_l-P_s)^3-(P_0-P_{01})^3]}{3a\eta_1(P_l+P)^3} - \frac{\nabla Tl}{T\left(\dfrac{l}{2}\right)} - \frac{\nabla Tl}{T\left(\dfrac{l}{2}\right)\ln\dfrac{P_l(P_0-P_{01})}{P_0[P_l-P_s(l)]}}\left\{\left(\frac{\eta_2-\eta_1}{\eta_1}+\frac{1}{a}\right)\left[\frac{P_{01}}{P_0}-\frac{P_s}{P_l}\right] + \right.$$
$$+\frac{(\eta_2-2\eta_1)\left[\left(1-\dfrac{P_s}{P_l}\right)^2-\left(1-\dfrac{P_{01}}{P_0}\right)^2\right]}{2a\eta_1} - \tag{18}$$
$$\left. -\frac{(\eta_2-\eta_1)\left[\left(1-\dfrac{P_s}{P_l}\right)^3-\left(1-\dfrac{P_{01}}{P_0}\right)^3\right]}{3a\eta_1}\right\} = -\frac{r^2(P_l^2-P_0^2)}{16\eta_1P_0D_0\left(1+\dfrac{\nabla Tl}{2T_0}\right)^m},$$

where P is the larger of the pressures P_0 and $P_s(\ell)$.

If we take $\eta(x) = \eta = \text{const}$ and $\nabla T = 0$, then Eq. (18) simplifies significantly:

$$\ln\frac{P_l - P_s(l)}{P_0 - P_{01}} = -\frac{r^2(P_l^2 - P_0^2)}{16\eta D_0 P_0}. \tag{19}$$

An equation differing slightly in form from Eq. (19) but identical in sense was obtained in [4]. In particular, the asymptotic behaviors of the function P_ℓ as $r \to 0$ in the present study and in (4) differ (it should be understood that the limiting transition is physically correct in view of the condition of continuity of the medium only to values $\mathrm{Kn} \sim 0.1$). It follows from Eq. (19) that P_ℓ is always less than the sum $P_s(\ell) + P_0 - P_{01}$ and tends to the latter value as $r \to 0$, while in [4] the pressure P_ℓ becomes larger than the sum $P_s(\ell) + P_0 - P_{01}$ with decrease in capillary radius. This difference is due to the fact that the present study considers the effect of change in pressure of the vapor–gas mixture along channel length upon the diffusion fluxes of the components. Just as in the present study, an estimate of the maximum value of P_ℓ was obtained by kinetic theory methods in [5].

It is of definite interest to delimit the ranges of existence of Stefan and viscous evaporation regimes. The transport regime is of the Stefan type if the liquid evaporates at a temperature below the boiling point from a capillary of radius sufficiently large that viscous resistance to the hydrodynamic flow of the mixture may be neglected. Constancy of the total pressure is assumed. If viscous resistance to the hydrodynamic flow is considered, then, as follows from Eq. (19):

$$P_l = P_0\left(1 + \frac{8\eta D_0}{r^2 P_0}\ln\frac{P_0 - P_{01}}{P_0 - P_s}\right). \tag{20}$$

With consideration of this, we write Eq. (13) for $\nabla T = 0$

$$j_1 = -\left[1 - \frac{8\eta D_0 P_s}{r^2 P_0 (P_0 - P_s)}\right]\frac{D_0 \mu_1 P_0}{RTl}\ln\frac{P_0 - P_{01}}{P_0 - P_s}. \tag{21}$$

Hence it is evident that the larger the capillary radius and the lower the evaporation temperature, the more accurately the vapor flux density is described by the Stefan expression, while the pressure of the vapor–gas mixture above the liquid meniscus is closer to the mixture pressure at the channel mouth. Consequently, the transport regime can be considered of the Stefan type, if the inequality

$$\frac{8\eta D_0 P_s}{r^2 P_0 (P_0 - P_s)} \ll 1 \tag{22}$$

is satisfied.

If the meniscus temperature is higher than the liquid boiling point, then $P_s(\ell) > P_0$, and, as follows from Eq. (18), as $r \to \infty$

$$P_l = P_s(l) + (P_0 - P_{01})\exp\left\{-\frac{r^2[P_s^2(l) - P_0^2]}{16\eta_1 D_0 P_0}\right\}. \tag{23}$$

In this case, from Eq. (13) we find

$$j_1 = -\frac{D(l/2)\mu_1 P_0}{RlT(l/2)}\left\{\ln\frac{P_s(l)}{P_0} + \frac{r^2[P_s^2(l) - P_0^2]}{16\eta_1 D_0 P_0}\right\}. \tag{24}$$

This expression differs from the well-known Poiseuille formula for viscous flow in its additional term in $\ln[P_s(\ell)/P_0]$. Therefore, the vapor transport regime can be considered viscous if the inequality

$$\frac{16\eta_1 D_0 P_0 \ln\dfrac{P_s(l)}{P_0}}{r^2[P_s^2(l) - P_0^2]} \ll 1 \tag{25}$$

is satisfied. The vapor flux is then described by the Poiseuille expression more accurately, the larger the capillary radius and the higher the liquid meniscus temperature.

In the case where neither of inequalities (23) or (25) is satisfied, the explicit form of the expressions for P_l, and hence, for j_1, cannot be obtained from transcendental equation (18), so that the dependence of vapor flux density on liquid miniscus coordinate is described only by systems (11) and (18).

NOTATION

x, current coordinate; ℓ, meniscus coordinate; T(x), vapor−gas mixture temperature; P_0, total mixture pressure at channel mouth; P_{01}, partial vapor pressure at channel mouth; $P_2(x)$, partial gas pressure; P(x), mixture pressure; $P_S(\ell)$, saturated vapor pressure above liquid meniscus; P_ℓ, vapor−gas mixture pressure above meniscus; ∇T, temperature gradient; μ_1, molecular mass of mixture; μ_2, molecular mass of gas; D, vapor diffusion coefficient in gas; ρ_1 and ρ_2, densities of vapor and gas; j_1 and j_2, density of vapor and gas fluxes; η, dynamic viscosity coefficient of vapor−gas mixture; L_0, molar heat of evaporation of liquid at temperature T_0; Kn, Knudsen number; r, capillary radius; $P_0 - P_{01}$, partial gas pressure at channel mouth.

LITERATURE CITED

1. N. B. Vargaftik, Tables on the Thermophysical Properties of Liquids and Gases, Halsted Press (1975).
2. E. L. Studnikov, "Viscosity of moist air," Inzh.-Fiz. Zh., 19, No. 2, 338-340 (1970).
3. A. N. Matveev, Molecular Physics [in Russian], Vysshaya Shkola, Moscow (1981).
4. M. N. Gaidukov, N. V. Churaev, and Yu. I. Yalamov, "Toward a theory of evaporation of liquids from capillaries at temperatures exceeding the boiling point," Zh. Tekh. Fiz., 46, No. 10, 2142-2147 (1976).
5. N. V. Pavlyukevich, G. E. Gorelik, V. V. Levdanskii, et al., Physical Kinetics and Transport Processes in Phase Conversions [in Russian], Nauka i Tekhnika, Minsk (1980).

DETERMINATION OF THE THERMAL CONDUCTIVITY AND THERMAL DIFFUSIVITY OF SOLIDS BY UNILATERAL SOUNDING OF THE SURFACE

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The article analyzes the possibility of determining thermophysical characteristics by nondestructive unilateral sounding of a flat surface. Curves are presented for selecting the optimum regime of the experiments in dependence on the dimensions of the sounded body.

The application of methods of nondestructive inspection of the thermophysical properties of materials under real conditions is impossible without taking into account the geometric dimensions of the body and its heat exchange with the environment. We will examine the effect of these factors on the result of measurement with the so-called isothermal sondes which are used in building, geology, refrigeration engineering [1-3]. Figure 1 shows a variant of the sonde for determining thermal conductivity λ and thermal diffusivity a of isotropic materials in the range λ = 0.03-10 W/(m·°K). The copper core with a flat circular contact area is surrounded by an adiabatic shell with a heater. The system of automatic temperature control SART-1 ensures that the temperatures of the core and of the shell are equal to each other, thus preventing heat losses from the core to the environment. The device is enclosed in a metal housing which can be shifted along the tubular supports. Before the experiment the sonde was mounted above the surface of the investigated body, and the core was overheated by ϑ_1 = 10-15°K relative to the initial temperature of the material, and then the sonde was lowered onto the surface. The magnitude of the overheating was measured by a battery of differential thermocouples, and with the aid of the regulator SART-2 it was maintained at a constant level during the entire experiment. Thermal conductivity and thermal diffusivity can be determined from the time dependence of the integral thermal flux proceeding from the core to the material [3]:

$$q(\tau) = 4r_0\vartheta_1\left[\lambda(1 - \mathrm{Bi}_1^{-1}) + 1.4r_0\frac{\lambda}{\sqrt{a}}(1 - 2\mathrm{Bi}_1^{-1})\ \frac{1}{\sqrt{\tau}}\right], \quad (1)$$

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